NAZARİYAT
Journal for the History of Islamic Philosophy and Sciences

Between Reality and Mentality -Fifteenth Century Mathematics and Natural Philosophy Reconsidered-*

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Abstract: Why did the members of the Samarqand Observatory School stand closer to the science of kalām for metaphysical principles in the fifteenth century and reserve more space to Mathematics in the description of the nature? When we look at the works circulating among scientists and emerging terms in this period, we observe some relative advancement in mathematical sciences used for quantitative certainty, also problematization of the ontology of mathematical entities and of epistemological values of mathematical knowledge, and discussions on the legitimacy of mathematical models on the nature. We examine the roots of these questions in Islamic tradition of philosophical sciences and especially developments post-Marāgha Observatory School; and analyze the posed ideas in relation to the concept of nafs al-amr (fact of the matter), which relies at the center of all research and discussions.

Keywords: Ontology of Mathematical Entities, Mathematical Knowledge, Relation Between Nature and Mathematics, Quantitative Certainty, Legitimacy of Mathematical Models, Fact of the Matter, Science of Kalām.

* I thank Jamil Ragep, Robert Wisnowsky, Reza Pourjavady and Sally Ragep who kindly shared thoughts and comments on the article's content, concepts and sources during my research and writing. I also thank two anonymous referees for their suggestions and proofreading. Without doubt, all mistakes belong to me.

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DOI http://dx.doi.org/10.15808/Nazariyat.1.1.M0001

Fazlıoğlu, İhsan, "Between Reality and Mentality -Fifteenth Century Mathematics and Natural Philosophy Reconsidered-", *Nazariyat Journal for the History of Islamic Philosophy and Sciences* 1/1 (November 2014): 1-39.

Mathematics was valued as a system for attaining knowledge and achieved a relatively uncontroversial place within premodern Islamic civilization[1]. Most of the learned class—whether among the Hellenized philosophers (*falāsifa*), the theologians (*mutakallimūn*), or the scholar-jurists (*fuqahā'*)—held mathematics in high esteem for either its high epistemological standing or its practical value. However, the application of mathematics to explain physical phenomena was far more controversial. This was mainly related to the ambiguity inherent in trying to relate mathematical entities to physical ones and in seeking to understand how mathematical proofs might provide an understanding of the physical world. For the Hellenistic-inspired philosophers of Islam, most prominently represented by Ibn Sīnā (Avicenna), the logical proofs based upon natural philosophical first principles provided a more secure understanding of nature than mathematical models, though there was the expectation that the two would be in conformity. In this the Hellenistic-inspired philosophers differed significantly from an early group of speculative theologians, the Mu'tazila, who argued that the physical world was based upon mathematical entities built from non-corporeal minimal parts. Hellenistic-inspired philosophers and the Mu'tazila thus held conflicting views concerning the role of mathematics in understanding the physical world. As a result, a long-term discourse about the nature of mathematical entities, their relation to physical phenomena and the status of mathematical proofs emerged among philosophers, theologians and scientists. Of primary importance for us here is the ongoing debate regarding the legitimacy of using mathematical models to explain the true nature of physical reality.

A related development occurred as a result of the criticism of some fundamental aspects of Avicennian philosophy, in particular the cognitive process of attaining knowledge through the active intellect. Especially after the severe attack of al-Ghazzālī, and the undermining -through reformulation- of Avicennian philosophy in the works of Fakhr al-Dīn al-Rāzī, it became critical to develop an alternative epistemology. Part of this process involved a new ontology of mathematical entities, a reformulation of the status of mathematical proofs, and a validation of mathematics as a way of understanding nature. The thesis of this essay is that these debates enhanced the status of mathematics as a legitimate means to understand the physical world and created a strong motivation to develop more precise mathematical and observational tools during the post-classical period of Islamic science.

1 The focus of the proposed ideas here and throughout the article is the *relationship* between the natural sciences and Mathematics. As it seen in the examples of Thābit ibn Qurra and Kūhī, the usage of Mathematics for some philosophical inquiries or the critique of Aristotelian philosophy through mathematics is subject of another research. In addition, the proposed ideas revolve around the *main-stream* Mathematics without taking into account several exceptions such as al-Kindī who adopted *handasi* perspective or Khalīlī and Khwarazmī who engaged in other sciences beside Mathematics. In short, the article focuses on a *kind*, not necessarily *individuals*.

Another critical consequence of this more privileged status of mathematics was the questioning of the assumption, held by both early "mathematical scientists" such as Ibn al-Haytham, and "natural philosophers" such as Ibn Sīnā, that the mathematical and physical approaches to nature could be reconciled, despite glaring contradictions that had been inherited from Hellenistic science. In insisting on the autonomy of the mathematical sciences, even when they were in principle reconcilable with natural philosophy, figures such as Quṭb al-Dīn al-Shīrāzī, himself an associate of the noted Avicennian Naṣīr al-Dīn al-Ṭūsī, began a process that would eventually lead to a declaration of independence from Aristotelian natural philosophy by 'Alī Qushjī of the Samarqand school and the opening up of the possibility of a new type of mathematically-based physics and cosmology. In parallel with this development was the increasing rejection by philosophers, theologians and scientists of a science mediated by the active intellect that would lead to knowledge that was universal, essential, and exact. Instead, the alternative epistemology relied upon human cognition, in spite of all its limitations in terms of observation, language, and abstract theory-formation, to reach truth/reality (*ḥaqīqa*). With such an enormous weight placed upon the human mind, and its perceptual and cognitive apparatus that had been found wanting from the beginnings of rational philosophy, it is no wonder that the Muslim intellectuals felt compelled to analyze the relationship between mental constructs to external reality. Here the Samarqand school's mathematical models of astronomy and the universe they were meant to configure provide an excellent case study for explicating this discussion.

An obvious question to raise in this context is whether the Islamic tradition that declared the autonomy of the mathematical sciences in their investigation of physical reality had an impact on similar (or parallel) scientific and philosophical developments in Europe that are often associated with the Copernican and later scientific revolutions. Much more research will be needed to deal with this issue. Our purpose here is to provide a sampling of original material from Islamic sources as a way of laying the foundation for future cross-cultural comparisons and investigation.

I. The Roots of the Question: Reality (*ḥaqīqa*) and Mental Constructs (*i'tibārāt*)

In his *Book of Proportions (Kitāb al-nisab)*, the Anatolian and Egyptian scholar Muḥammad al-Kāfiyaji (d. 879/1474) deals with proportion as a category of human knowledge.[2] Kāfiyaji classifies "proportion" as an autonomous scientific field, which

2 For more on Kāfiyajī, see F. Rosenthal, "al-Kāfiyajī," in *Encyclopaedia of Islam* 2nd ed., vol. 4, p. 414b.

he duly calls "the science of proportions (*'ilm al-nisab*)." In his book, he states the following:

ولذلك قيل: لو لا الإعتبارات لبطلت الحكمة. ومصداق ذلك علم الهيئة مع أن مباحثه مبنية على الأمور الإعتبارية والدوائر الموهومة. وإنما يعرض الخطأ على العقل في ذلك في بعض الأوقات لقصور استيلائه عليها بسبب من الأسباب. فلو لا اعتبار الأمور الوهمية والخيالية في بعض المقامات لما كان علم الهندسة من أصدق العلوم وأقواها، لأنه مبني على الحسيات المختلطة بالوهميات والخياليات. وقد قال العلماء: إن من الجملة اليقينيات القضايا الوهمية في المحسوسات.

And for this reason it is said: if not for mental constructs (***al-i'tibārāt***), philosophy would be invalidated. The corroboration of this [comes from] the science of astronomy (***'ilm al-hay'a***), whose investigation nevertheless is based upon mental constructs and conjectured circles. However, error does sometimes occur to the intellect regarding this [matter] due to its [the intellect's] incapacity to control them (i.e. mental constructs [***al-i'tibārāt***]) for one reason or other. For were it not for the mental construction of conjectured and imagined things in some situations, the science of geometry would not be among the most reliable and solid of the sciences; for it is built upon sensations combined with conjectured and imagined [things]. The learned have said: conjectured propositions are among the sensible certainties.[3]

In a marginal note he added in a manuscript copy of his book, Kāfiyaji emphasizes that he applies the term mental construct (*al-amr al-i'tibārī*) to "anything that does not exist in external reality."[4] He goes on to define the concept "the quiddity of a mental construct" as "that which has no external reality" and divides it into two ontologically and epistemologically distinct categories: one category is that "dependent on conjecture and imagination" (*bi-ḥasab al-wahm wa-al-khayāl*) and the other "dependent on the intellect" (*bi-ḥasab al-'aql*). Furthermore, Kāfiyaji notes that some scholars take as part of certain knowledge those conjectural propositions related to sensible things. Regarding this latter point, which he seeks to clarify, he explains why one needs to divide mental constructs into the two following groups:

أحدهما محتاج إليه كاعتبار الوهم في الحسّيات واعتبار العقل أموراً باستخدامه الوهم فيها مع استيلاء حكمه عليه في الأمور العقلية. ونوع اعتبار له عقلاً كالوهميات الكاذبة كأنياب أغوال؛ ويعلم كذبه بمخالفة حكمه بأمر حكم العقل به...

One [type of ***i'tibār***] is needed for a mental construct by conjecture for sensations and [also] for a mental construct by the intellect for intellected things through its employing con-

3 Kāfiyaji, *Kitāb fī al-nisab*, f. 3a.

4 Ibid., f. 2b (margin).

> jecture on them with the controlling judgment [of intellect] over [conjecture]. A [second] type of mental construct [*i'tibār*], which is by intellection, is, for example, false conjectures, like ghouls' fangs. One knows it is false by the contrary judgment of the authority of the intellect's judgment over it.[5]

Note that Kāfiyaji has placed mental constructs of sensations that are arrived at by conjecture [*wahm*] in the same category as mental constructs of matters in the intellect that are also arrived at by conjecture. What distinguishes them from the second category of mental constructs (i.e. false conjectures) is that the former have been vetted, so to speak, by the controlling judgment of intellect and have been deemed true.

In the analysis of the ontology of mathematical entities and the validity of mathematical entities pertaining to nature in Kāfiyaji's treatise, two conceptual issues stand out. One consists of presenting the constituents of human cognition-the sensible (*maḥsūs*), the conjectural (*mawhūm*), the imaginative (*mutakhayyal*) and the intelligible (*ma'qūl*)-and their interrelationships as a way to provide the basis of an ontology of mathematical entities. The other is an examination of the validity of mathematical knowledge as it relates to "truth" (*ḥaqīqa* or *ṣidqiyya*), which for Kāfiyaji is immanent in the term "mental construct" rather than in Ibn Sīnā's active intellect. I shall come back to these two issues later on in my discussion.

Whose philosophy of thought did Kāfiyaji's ideas challenge? Or to put it another way, in what context did Kāfiyaji formulate them? Answers to these questions shall provide a better understanding of Kāfiyaji's two conceptual issues mentioned above. For the purpose of elaborating on them and understanding Kāfiyaji's intention in formulating his ideas, let us turn to the ideas of 'Aḍud al-Dīn al-Ījī (d. 756/1355), as articulated in his very influential theological work, *al-Mawāqif fī 'ilm al-kalām*, and their criticism by al-Sayyid al-Sharīf al-Jurjānī (d. 816-1413), who played an important role at Tīmūr's court. As in many works in later kalām, Ījī takes the opportunity to discuss mathematics and the mathematical sciences in *al-Mawāqif*. After describing the basic geometrical objects, he goes on to state:

وهذا كلّه أمور وهمية لا يعلم وجودها خارجاً، وعليها مبني علمهم الذي يدعون فيه اليقين.

> These are all conjectural matters whose existence is not known in the external world [reality]. Upon them is founded their science, which they claim to be certain.[6]

5 Ibid., ff. 3a-3b.

6 al-Ījī, *al-Mawāqif*, p. 160. Two articles that have discussed this and related passages are Sabra, "Science and Philosophy in Medieval Islamic Theology", 1-42 and F. Jamil Ragep, "Freeing Astronomy from Philosophy", 49-71.

In Ījī's opinion, it is arguable whether or not such mathematical entities exist as such in the external world, i.e. in reality. For him, they are merely mental constructions. A second issue is whether or not sciences based on mental constructions alone can make a claim to certainty. Here, Ījī takes astronomy-an essentially mixed science that stands historically at the intersection of the mathematical and the natural-as a case in point. He provides a brief discussion of geometrical and astronomical terms and then concludes:

> فهذه أمور موهومة، ولا وجود لها في الخارج، ولا حجر في مثلها، ولا تتعلّق باعتقاد، ولايتوجّه نحوها إثبات وإبطال. إلّا إنّا أوردناها لتقف على مقصدهم، وإذا رأيته محض تخيّلات أوهن من بيت العنكبوت، لم يهلك سماع هذه الألفاظ ذوات القعاقع.
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> These are conjectural matters: they do not have external reality; there is nothing comparable to them; they are not related to conviction [*i'tiqād*]; and they are not subject to affirmation and refutation. We have only set them forth in order that you understand their intentions. And when you see them as purely imaginings, with less strength than a spider's web, hearing these clattering expressions will no longer frighten.[7]

Taken in conjunction with his previous statement, Ījī's remarks here suggest that the geometrical models of astronomy are ontologically mental. They also emphasize that these mental entities do not have any external reality nor do they correspond to anything existing in external reality. It is worth noting Ījī's use of the word *i'tiqād*, which we have translated as conviction. *I'tiqād* would normally mean religious belief. But here Ījī most likely uses it to mean the content that an affirmative proposition points to; indeed, in the classical tradition of philosophy in Islam, knowledge is referred to as *certain belief pertaining to reality* [*al-i'tiqād al-jāzim al-muṭābiq li-al-wāqi'*].[8] This interpretation is further strengthened by Ījī's insistence that "they are not subject to affirmation and refutation," which would not be the case if Ījī were thinking of religious belief. Thus in Ījī's view, based on the Avicennian tradition, mathematical entities and models cannot in themselves be knowledge (*'ilm*), which provides cause and form/quiddity (*ṣūra*), since they do not exist as such in external reality.

A demonstration that produces scientific knowledge would here point to the correspondence of a given proposition and fact (*ṣidqiyya*); invalidation represents their non-correspondence (*kidhbiyya*). Thus in Ījī's view, mathematical models are not subject to affirmation and refutation since they do not correspond with real-

7 *al-Mawāqif*, p. 207.

8 al-Maghnīsāwī, *Mughnī al-ṭullāb*, pp. 231-235 .

ity. In other words, we can interpret Ījī's statements from a logical point of view. Correspondence (*ṣidqiyya*) or non-correspondence (*kidhbiyya*) are not relevant to mathematical terms or models, which are conjectural concepts (*umūr wahmiyya*), inasmuch as they cannot be negated nor affirmed and they do not exist in reality. Moreover, Ījī stresses that declaring entities of this sort to be purely imaginative and more tenuous than a spider's web would resolve the matter. He borrows the metaphor "more tenuous than a spider's web (*awhan min bayt al-'ankabūt*)" from the Quran. In Sūrat al-'Ankabūt (29: 41), this metaphor stands for that which awaits those who take as patrons or protectors (*awliyā'*) someone other than the one true God. However, particularly after Fakhr al-Dīn al-Rāzī (d. 606/1209), the theologians attributed a logical meaning to this phrase, and applied it to things not related to demonstrative proof.[9] Thus the import of Ījī's statements come to light: mathematical models alone do not give demonstrative knowledge with respect to true reality (*min jihat al-ḥaqīqa*).

Just as we did with the case of Kāfiyaji, we should examine the context in which Ījī's ideas were formulated and ask against whom and against what ideas did Ījī formulate his own position. It is our contention that Ījī's views posed a challenge to the search for a mathematical cosmology, one that claimed to represent reality. The following passage from Quṭb al-Dīn al-Shīrāzī (d. 710/1311) illustrates the type of position we take Ījī to be reacting against:

فإني قد كنت برهة من الزمان عازماً على أن أحرّر لنفسي ولسائر الأخوان في علم الهيئة التي فاز بالسعادة عالمها وانغمس في الشقاوة جاهلها لكونه أشرف العلوم لأنّ شرف العلم [١] إما بكون معلوماته ثابتة باقية غير متغيّرة [٢] أو بكون الطرق المؤدّية إليها طرقاً يقينية مبرّأة عن شوب الظنون [٣] أو بكثرة فوائده وهذا العلم الذي نحن بصدده قد اجتمع له الفضل من هذه الجهات كلّها لثبات موضوعاته على أحسن نظام وأتمّ دوام على ما لا يخفى وكثرة فوائده على ما لا يحصى ووثاقة براهينه لكونها عددية أو هندسية لا شك فيهما بخلاف براهين الطبيعي والإلٰهي ولهذا لم يرج اتّفاق الحكماء فيهما وفاقت هي أمثالها من الفنون الحقيقية وعلت أشكالها من العلوم الحكمية ... إذ ليس علماً يتغيّر بتغيّر الأديان ويختلف باختلاف الزمان والمكان بل هو كموضوعه ثابت أبداً وأزلاً لا يستحيل ولا يقبل خللاً وكبراهينه القطعية يكون دائماً معقولاً إلى أن يقضي الله أمراً كان مفعولاً وككثرة منافعه وأقلّها الدخول تحت قوله عزّ قائلاً الذين يذكرون الله قياماً وقعوداً وعلى جنوبهم ويتفكّرون في خلق السموات والأرض ربّنا ما خلقت هذا باطلاً يكون سرمداً حاصلاً.

9 al-Mollā 'Idhārī, *Ajwibat li-i'tirāḍāt al-fāḍil mawlānā Luṭfī*, Istanbul, Süleymaniye Library, Şehid Ali Paşa MS 2829, ff. 34a-36a on 36a.

> I resolved at one time to compose for myself and for all colleagues a treatise in astronomy [*'ilm al-hay'a*] whose cognoscente gains happiness while its ignoramus is plunged in distress because it is the most noble of the sciences. For the nobility of a science is either due to its body of knowledge being fixed, permanent, and unchangeable; or due to the methods producing [this knowledge] being certain, free of any taint of doubt; or due to the multitude of its benefits. This science with which we are concerned has brought together priority in all these aspects: on the fixity of its subject matter according to the best system; the most absolute permanence as is obvious; and the multitude of its benefits that are innumerable. And its proofs are solid due to their being numerical or geometrical, about which there is no doubt, in contrast to the proofs of natural philosophy and theology [metaphysics]; for this reason agreement among the scholars of the latter two cannot be hoped for. And [its proofs] have surpassed their like in the positive sciences and its propositions excel in the philosophical sciences ... For it is not a science that changes with a change of religions, or varies over time and place; rather, it is like its subject-fixed permanently and unendingly, unchanging and immune to imperfection-and like its definitive proofs-always intelligible until God decrees [another] effective order-and like the multitude of its benefits. And [even] its most minor part has a place in the statement of the Almighty: "Whoever-standing, sitting or reclining-recall God and reflect on the creation of the heavens and the Earth [will say]: Our Lord! Thou hast not created this in vain" [Qur'ān, III.191], it is eternally existent.[10]

These words of Shīrāzī, which reflect Ptolemy's introduction to his *Almagest*,[11] are remarkably plain. Elsewhere in the *Nihāya*, Shīrāzī states the conventional view that the principles of astronomy are derived from metaphysics, natural philosophy, and geometry.[12] But what is significant here is that Shīrāzī is creating a hierarchy for the value of proof in different disciplines that privileges mathematics. As far as I know, this had not been articulated by earlier Islamic astronomers and philosophers, even those who acknowledged the exactness of mathematics. Shīrāzī's connection to the Marāgha Observatory and involvement in Ishrāqī (illuminationist) philosophy would, as we discuss below, have been influential in his articulation and defense of a "mathematical philosophy." Despite Shīrāzī's prestige, and the distinguished pedigree of his ideas, they did not escape criticism from both theologians and followers of Ibn Sīnā, reflecting the profound impact they had in Anatolia and Iran. Indeed, a reader of the passage just quoted felt compelled to object to Shīrāzī's glorification of astronomy by writing in the margin: "except for metaphysics."[13] And furthermore, in addition to Ījī's views that we have outlined above, Shams al-Dīn al-Bukhārī (fl. second half 14th century), who himself was the student of one of Shīrāzī's students, namely Quṭb al-Dīn al-Rāzī (d. 766/1365), was critical

10 Shīrāzī, *Nihāya al-idrāk*, f. 1b-2a.
11 Toomer, *Ptolemy's Almagest*, pp. 35-37.
12 Shīrāzī, *Nihāya*, f. 3a.
13 Ibid., folio 1b.

of Shīrāzī's views. Strikingly, Bukhārī expressed his criticisms in his *Commentary* on Najm al-Dīn al-Kātibī's (d. 675/1276) *Ḥikmat al-'ayn*, which also included Shīrāzī's gloss on it.

> ولم يبحث عن الرياضي إلا عن نبذ من الهيئة لما قاله صاحب المشارع والمطارحات من أن أكثره يبتنى على الأمور الموهومة والإعتبارات الذهنية. والمهمّ هو البحث عن أعيان الموجودات، ولهذا لم يبالغ الشيخ الرئيس في العلم الرياضي كما بالغ في الإلهي والطبيعي.
>
> [In his work, Najm al-Dīn al-Kātibī] did not investigate the mathematical [sciences] except for a small part of astronomy due to what the author of ***Kitāb al-Mashāri' wa-al-Muṭāraḍāt*** [i.e. Shihāb al-Dīn al-Suhrawardī] states, namely that most of these [sciences] are based on conjectural concepts and mental constructs. However, what is imperative is to inquire into the actual nature of existing things (a'yān al-mawjūdāt). For this reason, the Grand Master, Ibn Sīnā, did not overly immerse himself in mathematical science as much as he did in metaphysics and natural philosophy.[14]

Evidently, Shams al-Dīn al-Bukhārī pursued a two-pronged approach in his indirect criticism of Shīrāzī. First, he refers to the ideas of Shihāb al-Dīn al-Suhrawardī (d. 587/1191), who was the founder of Ishrāqī philosophy, which served as the basis of Shīrāzī's own philosophy. Second, he refers to the ideas of Ibn Sīnā (d. 428/1037), the towering figure of Islamic philosophical thought, who is here portrayed as being rather lukewarm toward the mathematical sciences.

Our discussion above of a number of texts written in the period from the thirteenth to fifteenth century-by Kāfiyaji, Ījī, Shīrāzī, and Bukhārī-points to the existence of a long-term dispute among philosophers, theologians and mathematicians over the ontological status of mathematical entities. This conversation is suffused with terms and ideas that are part of the legacy of Ibn Sīnā. This being the case, we now need to examine his ideas regarding the ontology of mathematical entities, which will allow us to better comprehend his stance on several crucial issues: the differentiation between truth and mental construct; the validity of mathematical knowledge in nature; and the relationship of mathematical knowledge and natural philosophy.

II. The Legacy of Avicennism

According to Ibn Sīnā, "philosophy" can be briefly defined as the acquisition of a specific sort of knowledge and acting according to this knowledge. In other words, philosophy teaches humans to understand the theoretical (*naẓarī*) capacity

14 Shams al-Dīn al-Bukhārī, *Sharḥ Ḥikmat al-'ayn*, p. 29.

of themselves and to know the truth (*ḥaqq*) relating to the reality of things (*ḥaqā'iq al-ashyā'*). They then can use their practical (*'amalī*) capacity to do good (*khayr*), thus eventually attaining perfection (*takmīl*). At the end of this process, humans achieve *happiness* (*sa'āda*). This moral-oriented definition, which unfolds its true meaning in the holistic character of ancient philosophical knowledge, is related to ancient theology and cosmology. This definition is also in some ways related to a certain theory of the soul, which is also closely linked to ancient theology and cosmology. In principle, the human soul, in the course of its cosmic journey, is supposed to actualize (*yaf'ala*) its potential (*quwwa*) to achieve truth and the good. In so doing, the human soul reaches its *entelecheia* (*kamāl*), thus fulfilling itself.[15]

"True knowledge (*ḥaqq, ḥaqīqa, 'ilm*)" and "good behavior (khayr)" are two key terms in this system. Therefore, Ibn Sīnā attempted, in many of his works, to introduce and explain the structure of human capacity to generate *true knowledge*. For the purposes of this paper, we may summarize this structure as follows:[16] the faculties of the soul that generate knowledge are ordered according to a cognitive analysis of knowledge. In this order, *human intellect* employs both outer and inner senses in order to engage nature. The processing faculty (*al-quwwa al-mutaṣarrifa*) synthesizes and analyzes the imagination (*khayāl*), which stores individual forms that originate externally, and memory (*ḥāfiẓa*), which also stores individual meanings originating externally. This processing faculty is called the imaginative faculty (*al-quwwa al-mutakhayyila*) when it is controlled by the conjectural faculty (*al-quwwa al-wāhima*). On the other hand, this processing faculty is called the thinking faculty (*al-quwwa al-mufakkira*) when it is controlled by the intellect (*'aql*). Without intellect, the imaginative faculty could only produce a distortion of external reality. Thus in order to generate certain knowledge of an existing sensible body, the intellect must engage in a process that includes not only an ascending aspect, which involves the five external senses and the internal faculties mentioned above, but also a descending aspect, which involves the *active intellect,* located in the sphere of the moon and containing quiddities and causes in their pure forms[17]. Thus, it is the human intellect, through a connection with the active intellect, that can restore

15 See Ibn Sīnā, *al-Madkhal*, p. 14; idem, *al-Ilāhiyyāt*, pp. 3-4. For a concise source providing this widespread definition of philosophy, see Ibn al-Akfānī, *Kitāb irshād*, p. 3. For how this definition functions in a general work of philosophy, see Shams al-Dīn al-Bukhārī, *Sharḥ Ḥikmat al-'ayn*, pp. 25-29.

16 In his analysis, Ibn Sīnā applies different terms for internal and external faculties of *nafs*. For example, see his *al-Nafs*, especially pp. 228-266; idem, *al-Najāh*, vol. II, pp. 5-23. In this paper, I have generally used the Avicennian terminology as it was codified in the post-classical period in works such as: al-Abharī (d. 663/1265), *Hidāyat al-ḥikma*, pp. 432-434 and Kātibī (d. 675/1276), *Ḥikmat al-'ayn*, pp. 147-150.

17 This is because the most significant quality of the active intellect is "the granter of knowledge" (*wāhib al-'ilm*). See Shīrāzī, *Sharḥ Ḥikmat al-ishrāq*, p. 13. According to Ibn Sīnā, the active intellect stands to the human self as the Sun stands to the eye. See Ibn Sīnā, *al-Nafs*, pp. 321-326.

what would otherwise be distorted by the imaginative faculty. The human intellect can then differentiate what is conjectural *(wahmī)* from what is sensible. Hence, the active intellect helps generate a correct representation. This being the case, during this process that is aided by the active intellect, it is possible to know the truth about things without being affected by imagination.

The conjectural faculty occupies a central position in the Avicennian system outlined above. One reason for this is Ibn Sīnā's rejection of earlier Mu'tazilite views regarding mathematical entities. Mu'tazilite theologians used Euclid's *Elements* to provide a foundation for their minimal parts, i.e. the smallest indivisible unit (*juz' lā yatajazza'* or *jawhar al-fard*) that formed the basis of their ontology and natural philosophy. Thus, they were depending on the presumed certainty and compelling evidence found in mathematics in general and geometry in particular. Consequently, this led Ibn Sīnā to revisit the ontology of mathematical entities and the validity of mathematical knowledge.[18] In doing so, Ibn Sīnā reviewed the cognitive constitution of knowledge and conducted a thorough inquiry in terms of the theory of soul (*nafs*), i.e. psychology, into how knowledge is generated and the stages through which this occurs. He emphasized that knowledge of nature was knowledge of its sensible attributes, referring to a natural body (*jism ṭabī'ī*) as a sensible body (*jism maḥsūs*). On the other hand, a mathematical body (*jism ta'līmī/riyāḍī*), a term used by Ibn Sīnā to distinguish this body from natural body, is realized by abstracting (*intizā'*) its quantitative attributes that are embedded in the categorical constitution of a natural body. Ibn Sīnā regarded the pure, abstract form of this body as an entity generated and represented by the human conjectural faculty. Thus, he called a mathematical body a conjectural body (*jism mawhūm*). His distinction between sensible body and conjectural body contributed a new dimension to the ontology of mathematical entities. At the same time, it limited the application of mathematics to nature. For a philosopher of nature could study mathematical body only insofar as it existed in natural body, in other words to the extent that its quantitative category allowed. Conjectural mathematical entities and their interrelations possess a kind of certainty inasmuch as they are free from matter. However, they cannot provide that which is intellected (*'aqlī*) because they cannot furnish cause, which is the criterion for understanding reality; nor can they provide us with knowledge of natural processes. We might then say that beyond sensible body (*al-jism al-maḥsūs*) and the conjectural body (*al-jism al-mawhūm*), there exists intellected body (*al-jism al-ma'qūl*), which one may say brings the two together. In Ibn Sīnā's view, an intellected body is the basis of *propter quid,* or reasoned fact (*limmī*), more so than the other two types of bodies.

18 See Rashed, "Natural Philosophy", pp. 287-307.

This framework can provide us with a deeper insight into the relationship between mathematical entities and natural philosophy in the system formulated by Ibn Sīnā. In Ibn Sīnā's view, we can conceive mathematical entities without associating them in the mind with definite matter. However, these mathematical entities should be associated with some kind of matter in order for them to exist in external reality. It is "some kind of matter," because the nature of geometrical objects does not require a particular matter. For this reason, the subject of a geometrical object is not determinate in terms of its species. That being the case, the definition of any geometrical object does not include its subject matter.[19] Unlike geometrical objects, natural objects have a definite matter related to its particular species in both the intellect and in external reality. For this reason, the definition of natural objects includes their defining matter.[20] In this regard, sciences that study mathematical entities are called abstract sciences (*al-ʿulūm al-intizāʿī*) since the mathematical objects they study are abstracted from matter. Mathematics conducts this abstraction through definition (*ḥadd*). Geometrical objects that come into existence by means of a definition are fixed by means of conjecture and remain unchanging within the conjectural faculty in actuality (*bi-al-fiʿl*); potentiality (*bi-al-quwwa*) is the attribute of that which exists as matter.[21]

Thus pure geometry (*al-handasa al-ṣirfa*) investigates geometrical objects determined by conjecture that have a constant, actual form. Accordingly, the subject of pure geometry is pure magnitude, a type of pure quantity. Now let us suppose that a mathematician examines magnitude as an attribute of sensible body, the latter being a subject of natural philosophy.[22] In other words, let us suppose that this mathematician looks into sensible body with respect to its quantity. In doing so, s/he will not be doing pure mathematics but rather one of the mixed sciences such as astronomy, optics, etc. that share the same subject with natural philosophy.[23] Therefore, we may suggest that the objects of pure geometry are conjectural, abstract, and not associated with definite matter, whereas a mixed science such as astronomy deals with an abstract quantity, i.e. magnitude, that is a natural circumstance associated with definite matter.[24] Thus even though astronomy and natural philosophy may share the same subjects, their aims are different. Astronomy, which is based on sensory observation and conjectural geometry, informs us that orbs are in this shape

19 Ibn Sīnā, *al-Burhān*, p. 129.
20 Ibid., p. 123.
21 Ibid., p. 118.
22 Ibn Sīnā, *al-Ilāhiyyāt* p. 22.
23 Ibn Sīnā, *al-Samāʾ al-ṭabīʿī*, pp. 41-42.
24 Ibid., pp. 42, 45.

and in that condition from a mathematical point of view. On the other hand, natural philosophy explains *why* orbs are in this shape or in that condition. According to Ibn Sīnā, astronomy, as a mixed science, combines *innī* (*quia* or assertoric) proof, with its reliance on mathematics, and *limmī* (*propter quid* or causal proof), with its reliance on natural philosophy.[25] All this means that pure mathematical constructions alone cannot give certain knowledge about nature in the absence of natural philosophy because such constructions would not include the cause. According to this Avicennian viewpoint, even when these constructions claimed to provide certain knowledge (and in fact did provide apodeictic knowledge), such knowledge failed the true test of scientific knowledge because it did not provide a cause and, being purely conjectural, could not correspond in an exact way to reality (*ṣidqiyya*). What is striking is the similarity of this "philosophical" viewpoint with the "kalām" stance of al-Ījī, which we discussed above.

III. From Paradigm to Perspectives

Ibn Sīnā's investigations into the role of human cognition for attaining truth were motivated by his wider epistemological interests.[26] And among the major outcomes of these investigations were downgrading the status of both the conjectural (*wahmī*) and the imaginary (*takhayyulī*) in relation to the intellected (*'aqlī*). As mentioned above, this subordinate status was meant as a direct rebuff to Mu'tazilite theologians. More generally, it also reopened questions of the nature of mathematical entities and the validity of mathematical knowledge in a way that challenged mathematicians who were committed to some form of Platonism. That some mathematicians had a different view of the matter can be inferred from a work by Ibn Sīnā's contemporary, the eminent scientist Ibn al-Haytham (Alhazen: d. ca. 431/1040)[27]. In his nonextant treatise, entitled *Maqāla fī anna al-burhān ma'nā wāḥid, wa-innamā yust'amal ṣinā'iyyan fī al-umūr al-handasiyya wa-kalāmiyyan fī al-umūr al-ṭabī'iyya wa-al-ilāhiyya* (Treatise on demonstrative proof being a single thing, despite it being used constructively [concretely] for geometrical matters but linguistically [abstract-

25 Ibid., pp. 42-43; Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy*, p. 107. Cf. Aristotle, *Physics*, II.2.

26 See Ibn Sīnā, *al-Ta'līqāt*, pp. 34, 35, 82: "Comprehending the truth in things is beyond human capacity. Whereas it is possible to know the characteristics (khawāṣṣ), necessities (lawāzim), accidents ('awāriḍ), and causes (asbāb), all of which belong to things, it is impossible to know the true differentia (*al-faṣl al-ḥaqīqī*)". "Humans can never know the truth in things because their source of knowing things is sense…".

27 For the question whether one or two Ibn Haytham existed in history, see. Sabra, "One Ibn al-Haytham or Two?", pp. 1-50. Personally, I consider the existence of a single Ibn Haytham who had tripartite processes of intellectual evolution.

ly] for natural philosophical and metaphysical matters),[28] Ibn al-Haytham evidently attempted to develop the notion of demonstrative (i.e. scientific) proof , so that it could include geometry, contrary to the opinion of Aristotelian/Peripatetic philosophers. As implied in the title of this treatise, Alhazen considers geometrical proof to be constructive (*ṣinā'ī*) or concrete, while Peripatetic philosophical proof is seen by him as linguistic (*kalāmī*) or abstract. But he maintains that both kinds of proofs can produce scientific knowledge. A similar defense of mathematics, at least in its capacity to provide scientific (*burhānī*) knowledge, would seem to have prevailed among mathematicians during the post-Avicennian period. Thus, and not coincidentally, 'Umar Khayyām (d. 525/1131) and the notable algebraist Sharaf al-Dīn al-Ṭūsī (d. 609/1213), who was either a student of Khayyām or a student of one of Khayyām's students, were among the earliest opponents of Ibn Sīnā's philosophy. Indeed, Sharaf al-Dīn al-Ṭūsī wrote a highly critical super-commentary on Ibn Sīnā's *Remarks and Admonitions* in which he challenged Ibn Sīnā's view that only intellected knowledge was scientific (*burhānī*) and defended the mathematicians' use of conjectural (*wahmī*) knowledge.[29]

Another challenge to Avicennian philosophy came from the Ishrāqīs (illuminationists) whose ontology conceived of the Universe in terms of geometrical magnitude (*miqdār*). The origins of this idea were contained in the *Kitāb al-Mu'tabar fī al-ḥikma* by Abū al-Barakāt al-Baghdādī (d. 547/1152), who, however, did not elaborate on it.[30] It was within this framework that Ishrāqī ontology could provide a framework for mathematicians to confidently construct their objects and knowledge based on those objects. As we indicated above, Quṭb al-Dīn al-Shīrāzī attributed a superior position to mathematical knowledge over natural philosophy and metaphysics, this being part of his overall Ishrāqī philosophy.[31] At the same time, the Ishrāqīs attempted to formulate an alternative to Avicennian epistemology, which required that the active intellect play a key role in acquiring certain knowledge. In contrast, the Ishrāqīs insisted on a direct contact between the human knower and external things, with both the outer and inner senses of the self being removed (*kashf*) from in-between the intellect and the existent. Obviously the active intellect's intermediation was likewise removed from consideration.

28 Ibn Abī Uṣaybi'a, *'Uyūn al-anbā'*, p. 556.

29 Mas'ūdī, *Kitāb al-shukūk*, f. 1a-121a.

30 Baghdādī, *Kitāb al-mu'tabar*, vol. III, pp. 196-209 [al-Faṣl al-'āshir: al-hayūlā wa-al-ṣūra].

31 It is noteworthy that Quṭb al-Dīn al-Shīrāzī wrote a commentary on *ḥikmat al-ishrāq*, the main text of Suhrawardī (d. 587/1191); this commentary became an essential source for Ishrāqī philosophy, superseding that of Shams al-Dīn al-Shahrazūrī (d. 697/1297-8). In addition, he consulted the works of Ibn Kammūna (d. 683/1284), who had been interested in the ideas of Suhrawardī; see Reza Pourjavady and Sabine Schmidtke, *A Jewish Philosopher of Baghdad*, pp. 28-35.

Their statement that "certain knowledge is acquired by direct contact, not by proof" means that witnessing (*'iyān*) eventuates when the "intellectual" eye (*'ayn*) touches on the being existing in actual reality (*al-wujūd al-'aynī*). It should be noted that the Arabic terms for witnessing, eye and actual are all from the same Arabic root *'ayn*, which means eye. That being the case, the Ishrāqīs, who called themselves the adherents of purification (*ahl al-tajrīd*), required the human intellect to be in direct contact with the "Light," which provided the way to unmediated reality. In contrast they called their Peripatetic opponents the adherents of "truth" (*ahl al-ḥaqīqa*), by which they meant those who were limited to language and the senses, both inner and outer. The Ishrāqīs also called their method *al-kashf wa-al-'iyān* (uncovering and witnessing), which they contrasted unfavorably with the Peripatetic method, which they called *al-baḥth wa-al-burhān* (predication and proof).[32] Nevertheless, and significantly, the Ishrāqīs did not completely reject the Peripatetic method of knowing "truth of individual things" (*ma'rifa*); rather, they emphasized that this method had certain limitations. Therefore, an Ishrāqī should pursue the Peripatetic methodology up to a point, at which point he needed to follow the Ishrāqī path.[33]

Unlike the Ishrāqīs, who stressed the formal character of various modes of knowing, Mutakallims were concerned with the underlying principles (*mabādi'*). While rejecting Ibn Sīnā's metaphysics, they could still embrace the technical content that each Peripatetic discipline produced regarding existing reality. As Ghazālī (d. 505/1111) put it in the *Mi'yār al-'ilm* (Criterion of the Sciences), what is important is not so much the technical content of mathematics, natural philosophy, and metaphysics, but rather their underlying principles and the dependence of their technical content upon them.[34] In this regard, Mutakallims, such as Ghazālī and more importantly Fakhr al-Dīn al-Rāzī, completely rejected the Avicennian version of the active intellect theory and its epistemological implications, but they could and did embrace the content of Ibn Sīnā's cognitive system, described above, based upon his analysis of the outer and inner senses. Thus they rejected any dependence of the cognitive faculty upon a direct external agent, but insisted that it was instead inherent in the self/soul. Furthermore, by disassociating the cognitive faculty from an external agent, these thinkers had provided the possibility for examining the cognitive faculty in its own right.[35] Thus Mutakallims accepted knowledge obtained

32 Shīrāzī, *Sharḥ Ḥikmat al-ishrāq*, p. 24.

33 For more on this point, see Shīrāzī, *Sharḥ Ḥikmat al-ishrāq*, pp. 2-5, 8, 11, 13-14, 21-26.

34 al-Ghazzālī, *Mi'yār*, p. 27.

35 Striking examples of this trend are provided by treatises on ethics that Mutakallims, such as 'Aḍud al-Dīn al-Ījī, wrote along the lines of Aristotelian philosophy. However, one should note that these treatises deal with the potentials of *nafs* in terms of their internal processes, not with respect to their cosmic connections; see his *al-Ahklāq al-'Aḍūdiyya* [=*al-Risāla al-shāhiyya fī 'ilm al-ahklāq*], Istanbul, Ragıp Paşa

by the senses (*ḥawāss*), conjecture (*wahm*), or intellect (*'aql*), contending that truth pertaining to reality is the result of a synthesis of all three.[36] This result, insofar as apprehending reality was concerned, was not absolute knowledge but was limited by the possibilities of human cognition. Knowing per se is not constant in terms of its method and content. Therefore, the human soul can develop different methods by which to know existing things. This helps us understand why the limitations on human knowledge, and its contingency, pushed the Mutakallims and others to develop alternative, i.e. non-cognitive, ways to know God; for they could not under any circumstances relinquish their belief in a single, personal, omnipotent, and volitional God.

In the post-Avicennian period, it was not only the Mutakallims, Ishrāqīs, and mathematicians who found Ibn Sīnā's solution to cognition problematic; even nominal Peripatetic philosophers claimed that, *pace* Ibn Sīnā, knowledge of an existent was a representation in the intellect that had been distorted by the outer and inner senses (i.e. *wahm* and *khayāl*, conjecture and imagination). Hence the correspondence of human cognition to an actual existent could not be guaranteed; moreover, even the active intellect would not be able to rectify this incongruity. Thus, they proposed that different methods were needed to know the truth about existents. In particular, in the second half of the 13th century, Saḍr al-Dīn al-Qunawī (d. 672/1274) directly quoted the following two sentences from Ibn Sīnā: "Comprehending truth in things is beyond human capacity," and "humans cannot know the truth in things at all because the source of human knowledge of existents is sense."[37] In his exchange of letters with his contemporary Naṣīr al-Dīn al-Ṭūsī (d. 672/1274), the Iranian reviver of the Avicennian system, Qunawī referred to these quotations as a way to show the deficiencies of the Peripatetic system.[38] However, the most illuminating passages on this issue were penned by Shams al-Dīn al-Bukhārī in his *Commentary* on *Ḥikmat al-'ayn*, which Najm al-Dīn al-Kātibī (d. 675/1276) had authored using an Avicennian framework. According to Bukhārī, the Avicennians sought to achieve certain knowledge (*'ilm al-yaqīn*) by the acquired intellect (*al-'aql al-mustafād*) through witnessing the intelligibles. But the mystic (*'ārif*) seeks certainty through two additional levels of knowing. The first is called the level of

Library MS 1428/28. For a commentary on this work written by Ījī's student Shams al-Dīn al-Kirmānī (d. 786/1384), see Süleymaniye Library, Hasan Hüsnü Paşa MS 744. This trend continued with commentaries written by Ṭāshkūbrīzāda in the 16th century (Süleymaniye Library, Şehid Ali Paşa MS 1547) and by Munajjimbāshī in the 17th century (Süleymaniye Library Ayasofya MS 2891).

36 See Ghazzālī, *Mi'yār,* pp. 29-33. Ghazzālī uses the following terms: *al-ḥākim al-ḥissī*, *al-ḥākim al-wahmī, al-ḥākim al-'aqlī*.

37 See footnote 26.

38 See *al-Murāsalāt*, especially pp. 52-53.

witnessed certainty (*'ayn al-yaqīn*) in which the adept is able to witness the intelligibles in their own separate World (*al-mufāriq*). The second is called the level of true certainty (*ḥaqq al-yaqīn*) in which the adept achieves unity with those intelligibles.[39] In fact, perfection (*kamāl*) in knowledge can be obtained only by ascending to one of these two levels. Clearly, then, a sage (*'ārif*) has the means to pass beyond philosophicalknowledge.[40]

To summarize, we can say that in Islam's post-classical period the attacks on Avicennian philosophy came from a number of quarters-mathematicians, Mutakallims, Ishrāqīs, 'ārifs/sages, and independent philosophers such as Abū al-Barakāt al-Baghdādī. We can add to this the flourishing astronomical research in the thirteenth century that came about due to the Marāgha observatory and increased the confidence of the mathematicians. As a result, the Avicennian system lost its status as the only coherent and complete cosmological, epistemological and psychological philosophy. A main reason for this decline was the fact that the active intellect-which was the ultimate guarantor and guide to the reality of things and to achieving good based on that knowledge-had lost its role and function even among the nominal adherents of the Avicennian system. It is our contention that, from the 13th century onward, the greatest transformation in Islamic intellectual history was the adoption of the idea of a multiplicity of ways to truth as opposed to a single way to truth.[41] This transformation marked a shift away from a system of paradigmatic philosophies and thought, which claimed to have obtained absolute truth[42], to a *perspective-or viewpoint*-oriented system of philosophy and thought, which merely claimed to have attained one aspect of truth but was willing to acknowledge the possibility of other aspects. Thus, a new understanding of truth and good emerged, considering them not to be single in existence but rather multiple in perspectives.

39 See Ibn Sīna, *Ithbāt al-nubuwwāt*, pp. 43-44. This intellect is also called as *al-'aql al-kullī* (universal intellect), *al-nafs al-kullī* (universal soul), and *nafs al-'ālam* (World soul).

40 Shams al-Dīn al-Bukhārī, *Sharḥ Ḥikmat al-'ayn*, p. 26.

41 Although Aristotle discusses multiple ways of acquiring truth (see, for example, his discussion in *Posterior Analytics* I.13), he establishes a "hierarchy" of truth in which only demonstrative knowledge (*episteme*) attains to absolute truth since it is necessary knowledge. In contrast, the perspectivists of post-classical Islam were more inclined to accept multiple ways of knowing that were not necessarily ranked.

42 On the notion of "complete knowledge" (*al-'ilm al-tāmm*) in his system, see Ibn Sīnā, *al-Ta'līqāt*, ed. 'Abd al-Raḥmān Badawī (Cairo: al-Hay'a al-Miṣriyyah al-'Āmma li-l-Kitāb, 1973; reprinted Qum, 1404 H.) For the invariability of knowledge pertaining to *cause* (*sabab*) see pp. 15, 23, 25. In reference to the completeness of his system, Ibn Sīnā claimed that he had determined eight hundred principles pertaining to the whole material world, including both the sub-lunar realm of the four elements and the celestial realm composed of aether.; see "Risāla fī al-ajrām al-'ulwiyya", p. 46.

This new approach to truth lessened the tensions among the traditionally antagonistic groups and helps us understand the rise of polymaths who were considered experts in the religious, philosophical and mathematical sciences simultaneously.[43] This lessening of tension, however, did not prevent each group from prioritizing its approach even while acknowledging the relative correctness of other approaches within this hierarchy. A clear consequence of this can be seen in philosophical discussions of the 15th century that include statements such as: "according to kalām" (*min qibal 'ilm al-kalām*), "according to philosophy" (*min qibal 'ilm al-ḥikma*), or even "according to kalām and philosophy (*min qibal 'ilm al-kalām wa-al-ḥikma*)." For example, Mu'ayyad-zāda (d. 922/1516), a student of Mawla Luṭfī (d. 900/1494) and Jalāl al-Dīn al-Dawwānī (d. 908/1502), introduced particular concepts by stating, "according to the principles of the philosophers" (*'alā uṣūl al-falāsifa*)" or "according to the principles of kalām" (*'alā uṣūl al-kalām*).[44] Unlike what one might expect in Ibn Sīnā's works, Mu'ayyad-zāda did not go on to attack the adherents of these positions but rather presented them as part of a kaleidoscope of truth.

During the 15th and especially the 16th centuries, this discourse reached a point where we can identify a new form of thought and writing, as exemplified in the works of Ibn Kamāl (d. 940/1534) and Ṭāshkūbrīzāda (d. 968/1561).[45] However, a theoretical framework of their new approach had already been visible in the classification scheme of al-Sayyid al-Sharīf al-Jurjānī in the early 15th century. In his glosses on Quṭb al-Dīn al-Rāzī's *Commentary* on *Maṭāli' al-anwār,* written by Sirāj al-Dīn al-Urmawī (d. 682/1283), Jurjānī undertakes to classify the methods that are pursued in acquiring the knowledge that will eventually lead to happiness. In his view, knowledge essentially seeks happiness, and one should know both the beginning of one's existence (*mabda'*) and the return or resurrection (*ma'ād*) in order to attain happiness. In addition, the ultimate purpose of all these endeavors is to

43 A famous example is Naṣīr al-Dīn al-Ṭūsī. Cf. Sabra, "The Appropriation and Subsequent Naturalization of Greek Science in Medieval Islam", 223-243 (reprinted in *id., Optics, Astronomy and Logic: Studies in Arabic Science and Philosophy* [Aldershot, Hampshire: Variorum, 1994], no. I, and in *Tradition, Transmission, Transformation*, eds. F. Jamil Ragep and Sally P. Ragep [Leiden: E. J. Brill, 1996], pp. 3-27).

44 Examples of this trend can be seen in the following works: *Majmū'a min rasā'il ḥawāshī mawālī al-Rūm fī baḥth al-jihāt min qibal al-kalām wa-al-ḥikma* (Istanbul, Süleymaniye Library, Ayasofya MS 2350) and *Hādhā al-kurrās yashtamilu 'alā as'ila mu'allaqa bi-al-'ulūm al-shattâ* (Istanbul, Süleymaniye Library, Halet Efendi MS 802, esp. Mu'ayyad-zāda's statement on f. 220b ['alā uṣūl al-falāsifa]).

45 Thus when we encounter scholars of this period discussing problems in philosophy and kalām, they often classify the solutions according to those given by "the philosophers", "the mutakallims" (the theologians) or "the 'urafā" (the Sufis). See, for instance, Ṭāshkūbrīzāda, *Risāla fī tafsīr ṣūrat al-ikhlāṣ*, Istanbul, Süleymaniye Library, Şehid Ali Paşa MS 9277, ff. 74b-82a. One of the best examples can be found in the treatise written by 'Abd al-Raḥmān Jāmī entitled *al-Durra al-fākhira fī taḥqīq madhāhib al-ṣūfiyya wa-al-mutakallimīn wa-al-ḥukamā' al-mutaqaddimīn*, edited by Nicholas Heer and 'Alī Mūsavī Bihbihānī (Tehran: Dānishgāh-i MakGīll / Mu'assasah-i Muṭāli'āt-i Islāmī Dānishgāh-i Tihrān, 1980).

reach knowledge of God. To do so, there are two methods that can be used: one theoretical (*naẓarī*), the other intuitional (*kashfī*). The theoretical is divided into two versions. The first is such that a revelation, from whatever source, is the basis of the system. In this case the thinker is a theologian (*matakallim*). In the second version, the system is theoretical but is not based upon a revelation. In that case, the thinker is Peripatetic, which, for Jurjānī, meant Avicennian. The intuitional method is also divided into two sub-categories. The first is based upon revelation. In this case, the practitioner is a sage (*'ārif*) or Sufi. In the second subcategory, revelation is not the basis. In that case, the practitioner is an Ishrāqī.[46]

By being based on revelation, what is meant is a system that takes God and the sacred text as the starting point. Philosophizing would then follow. If the system is not based on revelation, then philosophizing would come first, and it is religion that would follow. After the 13th century, this fundamental difference between kalām and philosophy could be and indeed was characterized by the dichotomy between those who conceived of God as the Volitional Omnipotent (*al-qādir al-mukhtār*) and those who maintained that He is the Necessary Being [*wājib al-wujūd*].[47]

IV. Al-Jurjānī's Synthesis

In tracing Kāfiyaji's conceptualization of reality and mental constructs, we have attempted to show the relevance of the notions of intelligible (*ma'qūl*), conjecture (*wahm*), and imagination (*takhayyul*), as well as the question of the validity of mathematics for knowledge of nature, from the writings of Ibn Sīnā to the late fifteenth century. Kāfiyaji displayed a certain confidence in articulating his ideas outlined above. To a large extent, the source of this confidence was provided by the writings of al-Sayyid al-Sharīf al-Jurjānī (d. 816/1413). Jurjānī was a major participant in the debates regarding the nature of mathematical entities and in fact criticized both the views of his predecessor al-Ījī and those of his erstwhile student Qāḍīzāda (d. after 844/1440).[48] In sharp contrast to Ījī, Qāḍīzāda espoused a perspective according to which mathematics provided the primary way to truth. Jurjānī critically examined both viewpoints and eventually proposed a middle view meant to resolve the tension between natural philosophy and mathematics. To begin with, in his com-

46 Jurjānī, *Ḥāshiya 'alā sharḥ maṭāli' al-anẓār*, ff. 10b-11a.

47 See Fahkr al-Dīn al-Rāzī, *al-Maṭālib al-'āliyah*, vol. 3, pp. 107-118 and Khōja-zāda, *Tahāfut al-falāsifa*, Istanbul, Topkapı Palace Library, Ahmad III MS 1927. The same dichotomy may even be found in popular books, such as Nev'i Efendi, *Natāyij al-funun,* edited by Ömer Tolgay (Divanyolu, Istanbul: İnsan Yayınları, 1995).

48 Ṭāshkūbrīzāda, *al-Shaqā'iq al-nu'māniyya*, p. 16.

mentary to Ījī's criticisms in the *Mawāqif* regarding the ontology of mathematical entities, Jurjānī responded as follows:

> وقد يقال: قامت البراهين على وجودها في مواضعها. وإن سلّم كونها أموراً وهمية، فلا ينافي ذلك كون أحكامها يقينية. ألا ترى أن العدد المركّب من الوحدات التي هي أمور اعتبارية أحكام صادقة بلا شبهة. ومن أنكر كونها يقينية، فقد كابر. وكذا الحال في المباحث الهندسية، يعلمها من يزاولها. فإن قيل: لا كمال في معرفة أحوال الموهومات. قلنا: إنّ الموهومات قد تكون عارضة في نفس الأمر للأعيان الموجودة. فيحصل لتلك الأعيان بسبب ذلك أحكام مطابقة للواقع. وقد يستدلّ بأحكام الأمور الوهمية على أحوال الأمور العينية. ولا يخفى شيء من ذلك على من له شعور ببراهين علم الهيئة من الحساب والهندسة.
>
> It may be said: proofs of existence occur in their proper places. And even if they are conceded to be [based on] conjectural concepts, this does not invalidate their judgments being certain. Don't you see that numbers, which are composed of units that are matters of mental constructs, are true judgments without any doubt? Whoever denies their certainty is [just] being stubborn. This is likewise the case in geometrical studies, as those who deal with them know. If it is said: there is no complete knowledge in knowing the situation of conjectural concepts, we say: conjectural concepts may occur as a "fact of the matter" (***fī nafs al-amr***) with respect to the actual nature of existing things (***al-aʿyān al-mawjūda***). So on account of that, judgments occur regarding those actual natures that are in accord with reality. One may infer the conditions of actual things by using judgments of conjectural concepts. None of this is unfamiliar to someone who is aware of the demonstrative proofs in astronomy based upon arithmetic and geometry.[49]

In another note in his commentary, Jurjānī discusses a passage of Ījī dealing with the nature of astronomical/mathematical models and with mathematical knowledge relating to nature. After giving a number of examples from astronomy, Jurjānī then proceeds with his main point:

> فهذه وأمثالها وإن لم تكن موجودة في الخارج، لكنّها أمور موهومة متخيّلة تخيّلاً صحيحاً مطابقاً لما في نفس الأمر، كما تشهد به الفطرة السليمة، وليست من التخيّلات الفاسدة كأنياب الأغوال، وجبال الياقوت، والإنسان ذي الرأسين. وينضبط بهذه الأمور أحوال الحركات في السرعة والبطء والجهة على الوجه المحسوس والمرصود بالآلات. وينكشف بها أحكام الأفلاك والأرض، وما فيها من دقائق الحكمة وعجائب الفطرة، بحيث يتحيّر الواقف عليها في عظمة مبدعها قائلا: «ربّنا ما خلقت هذا باطلاً.» وهذه فائدة جليلة تحت تلك الألفاظ، يجب أن يعتنى بشأنها، ولا يلتفت إلى من يزدريها بمجرّد العصبية الباعثة على ذلك. والله المستعان على كلّ حال.

49 Ījī, *Kitāb al-Mawāqif bi-sharḥ ʿAlī ibn Muḥammad al-Jurjānī*, vol. 2, p. 185.

> These and their like, even if they do not exist externally [i.e. outside the mind], yet they are conjectural concepts that are correctly imagined, corresponding to what is in the fact of the matter [*fī nafs al-amr*] as attested by sound instinct [*al-fiṭra al-salīma*]; they are not false imaginings such as ghouls' fangs, ruby mountains and two-headed men. By means of these [astronomical] notions, the conditions of [celestial] movements are regularized in regard to speed and direction, as perceived [directly] or observed with [the aid of] instruments. [By means of these notions also] discovery is made of the characteristics [*aḥkām*] of the [celestial] orbs and the earth, and of what they reveal of subtle wisdom and wondrous creation-in such a way that whoever apprehends them is awed by the glory of their Creator, [prompting] him to say: "Our Lord, thou has not created this in vain." This then is a valuable lesson that lies hidden in those words [of the astronomers] and that ought to be cherished, while ignoring whoever is driven to disdain them by mere prejudice. And God is He from Whom assistance is sought in all circumstances.[50]

One could elaborate on various aspects of Jurjani's remarks, but for the purposes of this paper we shall limit ourselves to the following points. According to Jurjānī, mathematical entities and models are conjectural and thus do not exist in external reality; nevertheless, judgments (*aḥkām*) based on them do conform to facts and events in external reality. Thus, the knowledge they provide pertaining to facts and events are certain. In fact, the above-mentioned mathematical models do exist in real terms inasmuch as they correspond to what is in objective reality [the fact of the matter: *nafs al-amr*], even though such models, as we shall discuss below, do not exist in external reality in a manner that can be perceived by the external senses.

'Alī Qūshjī (d. 879/1474), a member of the Samarqand Observatory, was well acquainted with the ideas of his teacher Qāḍīzāda as well as with those of Jurjānī and his predecessors. In his *Sharḥ al-jadīd 'alā al-tajrīd*, a commentary on Ṭūsī's famous kalām work, Qūshjī tries to clarify the relationship of mathematical models to physical reality on the basis of terminology. While discussing whether such terms as point and surface have external existence, he refers to Fakhr al-Dīn al-Rāzī's *al-Mulakhkhaṣ* and notes: "The Imām [i.e. Rāzī] denied that unit, point, and relationships (iḍāfāt) are things that exist externally." He then adds:

> أقول: إنّ البديهة لا تفرق في ذلك بين الأمور الموجودة في الخارج والاعتبارية الموجودة في نفس الأمر؛ فكما جاز في الاعتباري أن يشغل كلّ المحلّ، لا بطريق السريان، جاز في الخارجي أيضاً ذلك....لأنّا نقول: لو سلّمنا أنّها اعتبارية فليست من الاعتباريات المحضة، بل من الاعتباريات التي في نفس الأمر؛ ومثل هذه الاعتباريات يتصوّر فيها الكون بعد أن لم يكن كالعمى يحدث في الشخص بعد أن لم يكن.

50 Ibid., vol. 2, p. 432.

> I say: [Our] immediate understanding [*al-badīha*] does not distinguish in these matters between the things existing in external reality [*al-khārij*] and the mental constructs existing in the fact of the matter [*fī nafs al-amr*]. Just as it is possible for mental constructs to take in all [individual] places, but not as an undifferentiated mixture, this is also possible in external reality … Because we say: even though we admit that they are mental constructs, they are not pure mental constructs but are mental constructs that are in the fact of the matter. This type of mental construct is [also] one by which one may imagine an existing thing that had not existed previously, such as blindness occurring in an individual who had not previously been [blind].[51]

In Qūshjī's view, the issue is not only a question of true correspondence (*ṣidqiyya*) resulting from the conformity of mathematical terms and models to facts and events in external reality; it must also involve the ontologically independent category of *nafs al-amr* [fact of the matter], which guarantees the certainty of mathematical knowledge regarding nature and the existence of mathematical entities and models. Besides Qūshjī, Muṣliḥ al-Dīn al-Lārī (d. 979/1571) elaborated on this issue. In his *Risāla fī mas'alat tanāhī al-ab'ād*, Lārī discussed it in the context of geometrical figures. He states:

> فإن قيل: المقدّمات التي حررتها بأسرها، وقرّرتها عن آخرها أحكام وهمية لا يعتدّ بها، ولا يوثق عليها في بيان أحوال الأعيان الموجودة. قلنا: الأحكام الوهمية في المحسوسات صادقة؛ وقد يلزم الأعيان الموجودة بحسب نفس الأمر أمور وهمية تستحيل انفكاكها عنها.
>
> If it is said: the premises that you have formulated in their entirety and established in toto are conjectural judgments that can neither be counted upon nor relied upon in explaining the actual nature of existing things (*al-a'yān al-mawjūda*).

We say [in response]: conjectural judgments regarding sensible things are valid (*ṣādiq*). The actual nature of existing things (*al-a'yān al-mawjūda*) may follow from conjectural concepts according to the fact of the matter [*bi-ḥasab nafs al-amr*]; disentangling the latter [conjectural concepts] from the former [actual nature of existing things] is impossible."[52]

Let me here give a brief summary of the main themes of this paper, which center on the ontology of mathematical entities and the validity of mathematical knowledge as it relates to nature. As far as Ibn Sīnā is concerned, pure mathematical entities in the mind, which are abstractions from natural bodies, are conjectural or imaginary. A mixed science such as astronomy, whose mathematical models are

51 'Alī Qūshjī, *Sharḥ tajrīd al-'aqā'id*, p. 139.

52 Lārī, *Risāla fī mas'alat tanāhī al-ab'ād*, f. 86a.

based on these conjectural mathematical entities, cannot in consequence provide certain knowledge concerning nature if it depends solely upon these mathematical models. As we mentioned above, Ibn Sīnā would therefore insist that astronomy can only provide certainty when coupled, or "mixed", with natural philosophy. Thus Ibn Sīnā and his followers distinguish between the terms *bi-ḥasab al-ḥaqīqa* (according to reality), which is reserved for natural philosophy, and *bi-ḥasab al-dhihn/i'tibār* (according to the mind/mental construct), which is reserved for mathematics.

In the language of classical logic, the core of this problem is, as the title of Kāfiyaji's text [*Book of Proportions*] illustrates, the relation (*nisba)* between subject and predicate. Whether the relation is real (*wujūdī*) or mental (*dhihnī*) will determine the type of knowledge the proposition will convey. If the relations are mental, the proposition only conveys the "fact" (*inna*); if the relations are both real and mental, the proposition will convey the "reasoned fact" (*limā*). Again, this will mean for the Avicennian that only propositions in natural philosophy, which give the "reasoned fact" (propter quid), can convey certain knowledge.

Here, we may ask the following question: if we assume the logical proposition that a given mathematical model corresponds to something in nature, what type of knowledge does this correspondence entail? As is well known, Avicennian logic calls true (*ṣādiq*) those statements that conform to reality (*ḥaqīqa*). But post-classical Islamic logicians coined a new phrase, *underlying truth* (also *ḥaqīqa*), to express the converse relationship between reality and propositions. Thus if we take mathematical models as propositions, and if they correspond to reality, then we can say that these models are true and give us true knowledge about reality. On the other hand, if we judge the propositions from the perspective of reality (*bi-ḥasab al-ḥaqīqa*), then the mathematical models cannot provide underlying truth since they do not exist in reality. In other words, according to post-classical theory, *certainty* emanates not from mental constructs (*bi-hasab al-i'tibārāt*) but from underlying reality (*bi-ḥasab al-haqīqa*), since certainty must conform to reality. Therefore, with respect to mental constructs, mathematical models give *merely truth (ṣidq)* not *underlying truth (ḥaqīqa)*. Writers who make this distinction often point to the case of the apparently broken spoon in water. There is truth *(ṣidq)* in the fact that we observe it as broken; however, the underlying reality is that it is not broken. A mathematical theory of refraction may help explain what we see, but it may need to be modified over time to account for better measurements, etc. What is not modified is the underlying reality that the spoon is not actually broken.

The texts and authors that we have cited indicate that the philosophical discourse that began with Jurjānī during the fifteenth century led to a new principle, namely that mathematical models as propositions could offer true knowledge per-

taining to reality (though not underlying truth) even when they were considered to be conjectural (*wahmī*). As illustrated in the works of Jurjānī and ʻAlī Qūshjī, as well as in the later works of Muṣliḥ al-Dīn al-Lārī, the notion took shape that a new, autonomous, ontological-epistemological principle should guarantee the soundness of mathematical entities and models. This is the category of *nafs al-amr* [fact of the matter] that we have mentioned previously. In addition, old concepts had to be revisited and redefined, especially those we have discussed above in the sections called "the Legacy of Ibn Sīnā." Despite the reliance on Avicennian terms, the references and definitions had changed markedly. It is for these reasons that concepts such as mental existence (*al-wujūd al-dhihnī*) and *nafs al-amr* became the subjects of heated debates during the 14th, 15th, and 16th centuries.

V. In Search of New Concepts and Nafs al-Amr

As pointed out earlier, the developments examined in this paper, and especially the declining role of the *active intellect* as a guarantor of *certain knowledge* in classical (i.e. Avicennian) epistemological systems, required that the concept of mental existence that had been used to define knowledge (*ʻilm*) be reexamined. It was due to these developments that the concept of *nafs al-amr* came to assume the role that the *active intellect* had played in the Avicennian system. In brief, the Avicennian tradition defines knowledge as "attaining the *form* of a thing in the intellect." *Form*, the key term in this definition, connotes *essence*. It is realized by the initial action of the faculties of the external and internal senses and the subsequent connection of the intellect, duly prepared, with the active intellect. The outcome is called *mental existence* (*al-wujūd al-dhihnī*), which is knowledge (*ʻilm*). Mutakallims from the early period of Islamic intellectual history had rejected the concept of mental existence because of its association with the active intellect as defined by the modified Aristotelianism of classical Islamic philosophy. But from the time of Fakhr al-Dīn Rāzī (d. 606/1209), this became a central concept of discussion and debate among the Mutakallims as well as among the philosophers. Since, as we have seen, the original notion of the active intellect was either rejected or of lesser significance in both post-classical philosophy and theology, the central epistemological role it had previously played urgently needed to be redefined; it is for this reason that we find it discussed so extensively from the thirteenth century onward. There were numerous works written on mental existence that not only took up this topic but also the related matters of external existence (*al-wujūd al-khārijī*) and *nafs al-amr.* For instance, Naṣīr al-Dīn al-Ṭūsī, Shams al-Dīn al-Kīshī (13th c.), al-Sayyid al-Sharīf al-Jurjānī, ʻAlī Qūshjī, Ṣadr al-Dīn al-Dashtakī (d. 903/1498) and his disciples, and Jalāl al-Dīn Dawwānī (d. 908/1502) and his disciples, as well as a considerable number of oth-

er thinkers in Samarqand and Istanbul, authored many of these works on mental existence. This discussion continued into the 16th century, when we find Ibn Kamāl and Ṭāshkūbrīzāda, both in Istanbul, writing separate books devoted to this topic. Consequently, the discussions these intellectuals generated, when integrated with the concept of *nafs al-amr*, made a profound contribution to the discussions on the truth-value of mathematical knowledge as well.[53]

As mentioned several times previously, another concept related to the topic of this paper is *nafs al-amr*, which literally means "the thing itself" or, more interpretively, "the fact of the matter." Although *nafs al-amr* as a term can be traced back to Ibn Sīnā, it took on a variety of meanings depending on the author, which makes a coherent historical account difficult. In the post-classical period, the views of Naṣīr al-Dīn al-Ṭūsī can provide a starting point from which to examine the development of this concept. In *Kashf al-murād*, which was a *commentary on the Tajrīd al-i'tiqād* written by his teacher Ṭūsī, Jamāl al-Dīn al-Ḥillī (d. 726/1325) tells the following anecdote:

> وسألته عن معنى قولهم: إنّ الصادق في الأحكام الذهنية هو باعتبار مطابقته لما في نفس الأمر؛ والمعقول في نفس الأمر إمّا الثبوت الذهني أو الخارجي، وقد منع كلّ منهما ههنا. فقال: المراد بنفس الأمر هو العقل الفعّال، فكلّ صورة أو حكم ثابت في الذهن مطابق للصور المنتقشة في العقل الفعّال فهو صادق وإلّا فهو كاذب.
>
> I asked him [Ṭūsī] about the meaning of their statement: "truth regarding mental judgments [*aḥkām dhihniyya*] is by way of a mental construct [*i'tibār*] that is in conformity with *nafs al-amr*. That which is [usually] understood by *nafs al-amr* is either a mental or an external existence [*thubūt*], but in this case he denied each one of them. For he replied: "What is meant by *nafs al-amr* is the active intellect; thus each form or judgment that is established in the mind that corresponds to a form embedded in the active intellect is true; otherwise it is false.[54]

Because various schools and thinkers defined *nafs al-amr* in different terms, it would be a daunting task to provide an exact definition of it. For example, it could refer to knowledge of God, divine knowledge, the first intellect, the active intellect, the location of ideas and so forth, a point made by Dāwūd al-Qayṣarī (d. 751/1350)

53 Sulaymān, "Ishkāliyyāt al-wujūd al-dhihnī", pp. 148-190. For a detailed study of the question on mental being and external being, see al-Rifā'ī, *Mabādi' al-falsafa al-Islāmiyya*, vol. 1, pp. 277-315. For an analysis of this concept both thematically and historically, Muṭahharī, *Durūs fī al-falsafa al-islāmiyya*, vol. 1, pp. 203-213.

54 Ḥillī, *Kashf al-murād*, p. 104. It is worth mentioning that Ṭūsī himself wrote a treatise on *nafs al-amr,* entitled *Risāla fī ithbāt al-'aql al-kull* (or *al-mufāriq*), on which there are at least 5 commentaries. The present author has edited the text and commentaries and will publish them shortly.

in his work entitled, *Maṭla' khuṣūṣ al-kilam fī ma'ānī fuṣūṣ al-ḥikam*.[55] Al-Jurjānī put *nafs al-amr* within his own kalāmic framework, and dealt with it in several independent texts, which had a profound impact on subsequent discussions of the subject.[56] Indeed, thanks to Jurjānī, we could well argue that *nafs al-amr* became one of the most essential terms of the 15th and 16th centuries. Thus, 'Alī Qūshjī, Khōja-zāda, Dashtakī, Dawwānī, Ibn Kamāl, Ṭāshkūbrīzāda, and almost every other major thinker living in these centuries authored works on *nafs al-amr*. An elaborated, and relatively straightforward definition of the term, is given by 'Alī Ṭūsī (d. 887/1482), a scholar who lived in Istanbul during the second half of the 15th century:

إنّ نفس الأمر معناه نفس الشيء في حدّ ذاته، على أنّ الأمر هو الشيء نفسه. فإذا قلنا: «الشيء كذا في نفس الأمر، كان معناه أنّه كذا في حدّ ذاته. ومعنى كونه كذا في حدّ ذاته، أنّ هذا الحكم له ليس باعتبار المعتبر، وفرض الفارض. بل لو قطع النظر عن كل اعتبار وفرض، فهذا الحكم ثابت له، سواء كان الشيء موجوداً في الخارج أو في الذهن. وأمّا كون الشيء كذا في الخارج، فمعناه أنّه كذا في وجوده الخارجي، أي وجوده الأصلي كما عرفت.

فنفس الأمر تتناول الخارج والذهن، لكنّها أعمّ من الخارج مطلقاً إذ كلّ ما هو في الخارج، فهو في نفس الأمر قطعاً دون العكس، وأعمّ من الذهن من وجه. إذ قد يكون الشيء في نفس الأمر لا في الذهن، بأن يكون في الخارج ولا يحصل في ذهن. وقد يكون في الذهن لا في نفس الأمر، كالكواذب. فالأشياء الغير الموجودة في الخارج، تكون في نفس الأمر متّصفة بالصفات. ولكنّ لمّا لم يكن لها تحقّق إلّا في الذهن، فاتّصافها أيضاً في الذهن.

The meaning of ***nafs al-amr*** is the identity of something in its essence, per se, the "***amr***" being the thing itself.

Thus if we say: this something is in ***nafs al-amr***, it means that it is thus in its essence, per se. The meaning of its being thus in its essence, per se, is that this judgment regarding it is not due to someone making a mental construct nor to someone putting forth an assumption; indeed, even if thought were cut off from every mental construct and assumption, this judgment [regarding the thing] would still be fixed whether the thing exists externally or in the mind. Concerning something being such and such externally, its meaning is that it is such and such in its external existence, i.e. in its fundamental existence as you have learned.

Nafs al-amr encompasses both the external and the mental. However, it is in absolute terms more general than the external since everything that exists in the external [world] is included in *nafs al-amr*, whereas the converse is not the case [i.e.

55 Qayṣarī, *al-Rasā'il*, p. 47.

56 Jurjānī, *Risāla fī taḥqīq nafs al-amr*, ff. 3b-4a.

not everything that exists in *nafs al-amr* exists in external reality]. Furthermore, *nafs al-amr* is more general than mind in one respect (*min wajh*), since something might be in *nafs al-amr* but not in the mind, in that something might be external but does not obtain in the mind. Or it might be in the mind but not in *nafs al-amr*, such as [in the case of] falsehoods. Things that do not exist in the external [world] might [also] be in *nafs al-amr* with certain characteristics. However, since they only have actuality in the mind, their characterization (*ittiṣāf*) is also in the mind."[57]

'Alī Ṭūsī thus suggests that the most significant characteristic of *nafs al-amr* is its independence from both human thought and external reality. Everything that exists in external reality also exists in *nafs al-amr* but not everything in *nafs al-amr* corresponds to something in external reality. Mental entities represent a case in point, because truths that exist in the mind (such as mathematical entities) exist in *nafs al-amr* but not in external reality. On the other hand, certain mental entities and judgments, such as falsehoods, do not exist in *nafs al-amr*. For instance, take the statement "the number five is even." This judgment does not exist in *nafs al-amr* despite the fact that it is a conceivable mental judgment. And conversely, a judgment that exists in nafs al-amr will continue whether or not the human mind is aware of or thinks about it. The 18th-century scholar and lexicographer al-Tahānawī notes that the proposition, "a thing exists in *nafs al-amr*" suggests two meanings:[58]

> وقد يقال: معنى كونه موجوداً في نفس الأمر أنّ وجوده ليس متعلّقاً بفرض اختراعي سواء كان متعلّقاً بفرض انتزاعي أو لم يكن، فالعلوم الحقيقية موجودة في نفس الأمر بكلا المعنيين، والعلوم الاصطلاحية المتعلّقة بالفرض الانتزاعي موجودة في نفس الأمر بالمعنى الثاني دون الأول.
>
> It might be said: the meaning of something having existence in ***nafs al-amr*** is that its existence is not connected with a created assumption whether [that something] be connected to an abstract assumption or not. Thus the true [or positive] sciences exist in ***nafs al-amr*** by way of both meanings [unconnected with created assumption or connected with created assumption]. The terminological [i.e. technical] sciences that are connected with abstract assumption exist in ***nafs al-amr*** by way of the second concept [created assumption] but not the first [unconnected with created assumption, i.e. external reality].

Tahānawī is being somewhat obtuse here but we can ascertain that he is attempting to distinguish between the sciences dealing with external reality, i.e. nature, and those dealing with abstract entities, such as mathematics. The upshot is

57 'Alī Ṭūsī, *Tahāfut al-falāsifa*, p. 231.

58 Tahānawī, *Mawsū'at Kashshāf iṣṭilāḥāt al-funūn wa-al-'ulūm*, vol. 2, p. 1720. Tahānawī points out that ancient/classical (*qudamā'*) logicians do not differentiate between the fact of the matter (*nafs al-amr*) and external reality.

that *nafs al-amr* contains the truths of both, without allowing false assumptions created by the mind.

Based on the above statements, we shall suggest that *nafs al-amr* was considered to encompass that which is true in both the external and mental worlds, thus being an objective world comprising ultimate reality.[59] In order to clarify and explain this point, we can draw a frame for the notion of *nafs al-amr* based on the texts and information and some other sources. The historical development of the notion of *nafs al-amr* (*fact of the matter*) in different schools and trends of thought and among several thinkers are the subject of my future research.[60] In this article I am going to show the role of this notion as an ontological basis for the mathematical models used in astronomy and its epistemological application for defining mathematical entities.

The existent has two kinds of quiddities: first is the quiddities that the intellect conceptualizes (*ya'tabir*) through assumptions and that otherwise have no reality (*taḥaqquq*). Being purely conceptual (*i'tibārī*) and suppositional (*faraẓī*), these quiddities neither can produce any scientific knowledge nor be the subject of any scientific judgment. In other words, these quiddities, because of not having any reality (mental or extra-mental), are unable to be the subjects of the true sciences (*'ulūm ḥaqīqīyya*), that is the sciences which search the true existent (*mawjūd ḥaqīqī*). In short, these quiddities have been described as follows: i. They are purely conjunction (*wahmī*); ii. There is no external origin (*mansha'*) for their extraction (*intizā'*); iii. They have no correspondence (*muṭābiqa*) with the external reality (*al-wujūd al-khārijī*); iv. They are opposed (*mukhālif*) to the external reality and the external reality is opposed to them;

The second group of quiddities includes those with extra-mental or mental reality. These quiddities exist regardless of being assumed or not. Since they exist they are called true existents (*mawjūdāt ḥaqīqī*) or *nafs al-amr*. The true quiddities are also divided in two groups: The external existence (*wujūd khārijī*), and the mental existence (*wujūd dhihnī*), such as relations (*iḍāfāt*), which they have existence within our mind. The main difference between the first quiddity and the second one is reality *(tahaqquq)*. In other words, it is plausible for the entities of the second group to be the subject of correspondence (*muṭābiqa*), where as for the entities of the first group it is impossible.

59 At first glance, this seems to resemble Popper's third world. But before drawing this conclusion, we would need to ascertain whether *nafs al-amr* encompasses human cultural artifacts, which are included in Popper's conceptualization. This will need to be a subject of future research.

60 İhsan Fazlıoğlu, *Humanizing knowledge: the theory of mental existence and nafs al-amr in Later Islamic Philosophy*, forthcoming book.

The question is, what is difference between suppositional and mental quiddities if both are mental? Suppositional quiddities do not follow the logical principals (*aḥkām manṭiqiyya*), but mental quiddities are produced by the act of extraction (*intizā'*) and abstraction (*tājrīd*) of the intellect. They are like mirror, shadow or attributes of a name. These sorts of quiddities can produce truthful knowledge. As mentioned above, while there is no external reality to which a suppositional quiddity could correspond, there is an actual and definite (*muta'ayyan*) entity with which the mental quiddity correspond. For that reason mental existence is true existence, which exists in the *nafs al-amr*. In this frame, we can summarize the basic features of mental existence as follows: i. They have been extracted from concrete reality; ii. They are not in contradiction with the reality; iii. They do not correspond to quiddities in the external reality; iv. Their correspondance with the external reality is judgmental; v. It is only in the *nafs al-amr* that they correspond with reality both in terms of quiddity and judgment.

We should emphasize on two consequence of this doctrine: (1) 'external' reality (*khārij*) could refer to two different things: external from supposition, in this sense it means true existence and external from mind which means the concrete world. (2) Not only external existence but also mental existence has true existence. For this reason, the *nafs al-amr* is more general than the external so that everything which exists in the external world exists also in the *nafs al-amr*; but the reverse is not true; i.e., everything mental does not have external reality in terms of correspondence with an external quiddity. Therefore, it is true to say everything which exists in the mind exists in the *nafs al-amr* as well (as a mental existence and not necessarily as an external existence). It is also true to say that the external quiddities are distinct (*mughayyar*) from their abstracted mental existence, which are called their resemble (*shabah*) or their shadow (*zıll*) are distinctive in terms of their consequences (*āthār*).

When we look at the meaning of *nafs al-amr* through correspondence, we face another problem: what is the difference between true and false propositions? In other words, if the corresponding term is problematic how can we distinguish true and false propositions? The solution is related to comprehending the reference of the term 'external.' The true proposition is the one which has correspondence in the external world and the false proposition is the one with no correspondence in external reality. The term 'external' here doesn't refer to concrete existence only. It means that it is out of supposition of the intellect. So, every proposition corresponds with true existence, either in the concrete world or in the mind. Having no correspondence in the concrete world does not rule out the correspondence with reality all together.

Thus, mathematical models, for instance, which have no correspondence in the concrete reality can still, be regarded as true existence. A circle, which is obtained through the process of extraction from circles-like things in the extra-mental world, does not exist in reality, yet by being mental, it exists in the nafs al-amr as a true existence. Judgments based on these quiddities correspond likewise in the external world and *nafs al-amr*. But things created by human supposition are there only during the supposition process. After ending of this action they leave the scene since they do not have essential actuality (*fi'liyya bi al-dhāt*) in the mind. For instance, supposing a human who has wings and flies. The supposition can sustain as long as the human's conjectural and imaginative faculties maintains this image. When this action ends, it disappears because it has not obtained any essential actuality in the mind.

VI. Natural Philosophy and Exactitude in Mathematics

As a result of the developments examined above, it came to be accepted among a significant group of post-classical scholars that mathematical entities have a correspondence with reality and that mathematical knowledge related to nature has truth-value (*ṣidq*). However, before mathematics could provide certainty about nature, one needed improved algorithms and new calculation methods. In fact, mathematical techniques and computational mathematics flourished and made marked advances during the 15th century. To place these developments within a larger context, we need to discuss another issue that contributed to the discourse on the validity of mathematical knowledge and the application of mathematical statements to nature. We may summarize this as follows.

The mathematical sciences could be seen to provide a more accurate way to fulfill certain commandments found the Quran, i.e. the Revealed Book (*al-Kitāb al-tanzīlī*), such as those regarding the obligatory shares of inheritance, prayer times, the beginning of the month of fasting (*Ramaḍān*), land surveying and so on. In other words, mathematics acts as an agent for understanding and executing a particular commandment or prohibition by God. Likewise, mathematical sciences could play the role of agency in the knowledge of the Universe, i.e. the Created Book (al-*Kitāb al-takwīnī*). In addition, applied geometry (*misāḥa*) and its applications to architecture provided a means to construct with greater precision sensible manifestations of religion (such as mosques, shrines, etc.).[61]

61 For an elaboration, see Fazlıoğlu, *Uygulamalı Geometrinin Tarihine Giriş*, Turkish introduction, pp. 1-96, Arabic text pp. 98-168.

But in order to adhere to these religious requirements, mathematics needed to be purged of its Pythagorean and mystical elements and to develop a new, formal, symbolic language that could be considered religiously neutral. The arithmetic, algebra and applied geometry that had been developed in the ninth century by al-Khwarizmī met these criteria. So by the 13th century, Ḥanafī scholars of Islamic jurisprudence such as Ismā'īl Mārdīnī (d. 637/1240) had considered them simply to be a means of calculation. Indeed, Mārdīnī wrote a treatise on number theory (*'ilm al-'adad*) that completely ignores the Pythagorean mysticism that one may find in Nicomachus of Gerasa or the Ikhwān al-Ṣafā'.[62]

During the 14th century, the school of Ibn al-Bannā' (d. 721/1321), a scholar of Andalusian origin, had developed advanced algorithmic calculation methods for fractions, exponents and roots, as well as algebraic notation and a system of symbols for equations.[63] Towards the end of the century, their findings reached Egypt thanks to the works of al-Qalaṣādī (d. 891/1486) and al-Ghāzī (d. 919/1523). Later in the 15th century, the Mamluk mathematicians Ibn al-Hā'im (d. 815/1412),[64] Ibn al-Majdī (d. 850/1447), and Sibṭ al-Mārdīnī (d. 912/1506) popularized through numerous publications Indian calculation (*al-ḥisāb al-hindī*), sexagesimal calculation (*al-ḥisāb al-sittīnī*), and in general algorithmic calculation methods.[65] By this time, Egyptian Shāfi'ī scholars of Islamic jurisprudence were following their Ḥanafī predecessors by extensively applying algebra in Islamic law. The development of these new calculation techniques led the astronomers in Egypt, who, in contrast to their colleagues further east, were less inclined to deal with the theoretical aspects of geometrical-kinematic models in astronomy, to depend mainly on numerical analysis in establishing an advanced numerical astronomy in the Egyptian-Mamluk region.[66]

In addition, the mathematical sciences in the post-classical period witnessed remarkable progress in the region that stretched from Central Asia to Anatolia through Iran, thanks to the impact of the Marāgha Observatory. Among the many members of the Marāgha Observatory, and their successors, we should mention Naṣīr al-Dīn al-Ṭūsī (d. 672/1274), the founder of the observatory, Quṭb al-Dīn al-Shīrāzī (d. 710/1311), Ibn al-Khawwām (d. 724/1324), Niẓām al-Dīn al-Nīsābūrī (d.730/1330), Kamāl al-Dīn al-Fārisī (d. 718/1329), and Jamāl al-Dīn al-Turkistānī (known to be alive in 712/1312).

62 Brentjes, "The First Perfect Numbers and Three Types", pp. 467-483.
63 Ibn al-Bannā, *Raf' al-ḥijāb*, pp. 19-38, 43-44, 77-90.
64 Fazlıoğlu, "İbnü'l-Hâim", pp. 62-65.
65 Fazlıoğlu, "İbn el-Benna", pp. 530-534.
66 King, "The Astronomy of the Mamluks", pp. 531-555.

The cumulative effect of all these developments helped the Samarqand mathematical-astronomical school advance calculational mathematics even further and to create more exact methods. In particular, the notion of *mathematical exactitude* during the 15th century led Ghiyāth al-Dīn Jamshīd al-Kāshī, the most important mathematical representative of the Samarqand School, to reinvent decimal fraction calculation and to put four basic arithmetical operations into more applicable forms. In addition, Kāshī made the most significant research into the number π since Archimedes and determined this number correctly to sixteen decimal fractional digits. He also calculated *sine 1°* with equal exactitude.[67] In fact, as J. P. Hogendjik notes, "In the determination of π, and in computational mathematics as a whole, al-Kashi was a pioneer."[68] Later on, the works of Ulugh Beg (d. 853/1449), Qaḍīzāda al-Rūmī and 'Alī Qūshjī[69] made additional contributions based on Kāshī's findings.

We may also note that a purely practical field such as accounting mathematics provided an entry point for Ottoman mathematicians into issues of exactitude. In particular, certain members of the Ottoman Imperial Accounting Bureau, in particular Khayr al-Dīn Khalīl (the latter half of the 15th century) and Ḥājjī Aṭmaja (known to be alive in 899/1493-1494), wrote texts dealing with accounting mathematics. The Bureau was modeled on its Īlkhānid predecessor, which had operated in Iran and Anatolia, and was founded in Istanbul during the second half of the 15th century with the purpose of dealing with imperial financial matters.[70]

The conception of mathematical exactitude as formulated in the Samarqand School found a receptive audience in this case, as well as others, thanks to relevant works authored in Istanbul. We may mention in particular developments in the 16th century due to Mīram Chelebī, (d. 931/1524) in the first half of this century and Taqī al-Dīn Rāsid (d. 993/1585) in the second half. Building on the work of his predecessors, Taqī al-Dīn successfully used decimal fractions in the calculation of exponential and rooted quantities and for the first time employed them for preparing astronomical and trigonometrical tables. He also used the mechanical clock to determine time with more exactitude and invented observational instruments to be used in preparing the tables for *zīj*es.[71]

67 Kāshī, *Miftāḥ al-ḥisāb*. On Kāshī's work on sine 1° (*Risāla fī istikhrāj jayb daraja wāḥida*), see Hogendijk and Rosenfeld, "A Mathematical Treatise Written in the Samarqand Observatory of Ulugh Beg", pp. 25-65.

68 Hogendijk, "al-Kāshī's Determination of π to 16 decimals in an Old Manuscript", p. 85.

69 'Alī Qūshjī, *al-Muḥammadiyya fī al-ḥisāb*, Istanbul, Süleymaniye Library, Ayasofya MS 2733/2.

70 Fazlıoğlu, "Osmanlı Klasik Muhasebe Matematik Eserleri Üzerine Bir Değerlendirme", pp. 345-367; and İhsan Fazlıoğlu, "Devlet'in Hesabını Tutmak: Osmanlı Muhasebe Matematiğinin Teknik İçeriği Üzerğ ine", pp. 165-178.

71 See Fazlıoğlu, "Taqī al-Dīn", pp. 1122-1123.

The conceptual developments analyzed above were not confined to the realm of practical results. These developments were also partially responsible for several new departures, especially during the 15th century. An important example was the new definition of number proposed to supplant the Egyptian, Euclidian, and Aristotelian definitions. The origins of this transformation in the definition of number, though elusive, certainly date back to a work on arithmetic by Jamāl al-Dīn al-Turkistānī.[72] Jamshīd al-Kāshī placed Turkistānī's definition at the core of his *Miftāḥ al-ḥussāb*,[73] and Muḥammad Shāh al-Fanārī (d. 839 H/1435-6 CE) states it as well in his *Unmūdhaj al-'ulūm*.[74] Later, 'Ali Qūshjī used it in his textbook entitled *al-Muḥammadiyya fī al-ḥisāb*.[75] This new definition eschews any metaphysical and theological speculation about numbers, in particular regarding the transcendent meaning of *one;* numbers are only considered insofar as they are the result of *counting operations*. Since such a definition excludes a mystical Hermetic-Pythagorean interpretation of numbers, we need to ask the following question: What was the philosophical or ideological framework that underlay this new mathematical approach in the 15th century? It is our contention that this mathematical framework can be characterized as *mathematical humanism* whose roots can be traced back to Platonism.[76]

The position of natural philosophy vis-à-vis the rise of mathematical humanism in the fifteenth century, as outlined above, was very complex. In part this was the result of the eclecticism, discussed above, whereby Peripatetic, Ishrāqī, and Kalāmic natural philosophies were often juxtaposed. But in the early 15th century, Peripatetic natural philosophy was in decline. For example, it was viewed less favorably in Bursa than approaches offered by kalām and 'irfān, and at the Samarqand School there was scant attention paid to it. In Samarqand the main texts dealing with natural philosophy were the kalām works of Taftāzānī (d. 792/1390) (*Sharḥ al-maqāṣid)* and Jurjānī (*Sharḥ al-mawāqif).*

At the Samarqand School, in which, according to Jamshīd al-Kāshī, more than one hundred mathematicians were studying, Peripatetic philosophy was stud-

72 Jamāl al-Dīn al-Turkistānī, *al-Risāla al-'alā'iyya fī al-masā'il al-ḥisābiyya*, Istanbul, Süleymaniye Library, Ayasofya MS 2729. There is new evidence showing that this definition was used previously by linguists working in the field of Arabic language. Ṭāshkūbrīzāda mentions this definition and refers it to the Arabic linguists (*arbāb al-'arabiyya*), saying that Muḥammad b. Ḥasan al-Astarābādī (d. 686/1287) gave this definition in his book entitled *al-Wāfiya fi sharḥ al-kāfiya fi al-naḥw* (Istanbul, Nuruosmaniye Library MS 2286; Berlin, Staatsbibliothek Sprenger MS 1823, f. 2b).

73 *Miftāḥ* (ed. Nabulsī), p. 47; *Miftāḥ*, (ed. Dimirdāsh and Shaykh), p. 44.

74 Muḥammad Shāh al-Fanārī, *Unmūdhaj al-'ulūm*, Istanbul, Süleymaniye Library, Hüsrev Paşa MS 482. Also see Fazlıoğlu, "İthâf'tan Enmûzec'e Fetihten Önce Osmanlı Ülkesi'nde Matematik Bilimler", pp. 131-163.

75 Qūshjī, *al-Muḥammadiyya fī al-ḥisāb*.

76 The author owes this idiom to Jamil Ragep.

ied mainly from Athīr al-Dīn al-Abharī's *Hidāyat al-ḥikma* and Mawlānā-zāda al-Harawī's *Commentary* (written in the first half of the 15th century) on Abharī's text. That 15th-century scholars found these texts, in addition to Najm al-Dīn al-Kātibī's *Ḥikmat al-'ayn,* sufficient for understanding Peripatetic philosophy gives us strong evidence that kalāmic and mathematical approaches to natural philosophy were much more prevalent in this century. However this neglect changed somewhat later in the 15th century. For example, Istanbul Ottoman intellectuals, who generally had a kalāmic perspective, nevertheless took parts of Avicennian natural philosophy as needed for certain subjects without seeking to comprehend or use the Avicennian system in a holistic manner With this in mind, it is rather striking that Khōja-zāda and 'Alī Ṭūsī each penned a *Tahāfut* [*Incoherence*] text that reformulated the much earlier controversy between the philosophies of Ibn Sīnā and Ghazālī. Commissioned by Sultan Mehmed II, these *Tahāfut*s drew upon insights obtained from centuries of debate and reflection by philosophers and mutakallims. Furthermore, significant arguments and texts related to Avicennian natural philosophy emerged in this period that spanned a broad geographical network connecting Samarqand, Iranian intellectual centers, and Istanbul. Some of the notable intellectuals engaged in these arguments and in producing texts included Jurjānī, Mawlānā-zāda al-Harawī, Ṣadr al-Dīn al-Dashtakī, Jalāl al-Dīn Dawwānī, Khōja-zāda (d. 893/1488), Khaṭīb-zāda (d. 901/1495), Sinān Pāsha (d. 891/1486), and Abū Isḥāq al-Nayrīzī (d. 884/1479). During the reign of Sultan Mehmed II and his son Sultan Bāyazīd II, the Ottoman Imperial Palace hosted scores of debates concerning cosmology and astronomy. Through these debates, participating intellectuals revisited questions regarding the ontology of mathematical entities and the truth-value of mathematical knowledge related to nature as well as the arguments of Peripatetic natural philosophy.[77]

In analyzing the tension between mathematics and natural philosophy in the 15th century, we need to keep in mind that contemporary scholars who were associated with one or the other of these fields-even those in the Samarqand School where mathematical humanism held a dominant position-were far from unified in their opinions. For instance, whereas Jamshīd al-Kāshī, Qaḍīzāda, Ulugh Beg, and 'Alī Qūshjī adopted more of a mathematical approach to natural philosophy, Fatḥullāh Shirwānī (d. 891/1486), 'Abd al-'Alī al-Birjandī (known to be alive in 935/1528), who had either studied at Samarqand or were taught by members of the Samarqand School, as well as others, adhered to a version of Peripatetic natural philosophy. On

77 See Istanbul, Süleymaniye Library, Halet Efendi MS 802; Hasan Hüsni MS 600; Ayasofya MS 2391 (for Abū Isḥāq al-Nayrīzī).

the other hand, Taftāzānī, al-Jurjānī, and the latter's students relied on kalāmic natural philosophy to a large degree. And while 'Abd al-Raḥmān Jāmī (d.898/1492) criticized Peripatetic and kalāmic natural philosophy from the point of view of 'irfānī mysticism, 'Ali Qūshjī was influenced by Ishrāqī arguments in formulating his criticisms of both the Peripatetic-philosophical and the Ash'arite-theological positions. Indeed, Qūshjī's criticisms were so radical that they even promoted the idea of stripping astronomy of Peripatetic physics and metaphysics entirely.[78] Ghulām Sinān (d. 911/1506), one of Qūshjī's students, embraced this idea.[79] But later, Qūshjī's own grandson Mīram Chelebī (d. 931/1524), would reincorporate Peripatetic and kalāmic natural philosophy into his studies as exemplified in his *Risāla fī qaws quzaḥ* on the rainbow.[80]

It is our contention that in Iran during the late 15th century and early 16th centuries, the affection of Ṣadr al-Dīn al-Dashtakī and his disciples for Avicennian philosophy was a reaction to the mathematical humanism embraced in Samarqand and the kalāmic natural philosophy followed in Istanbul. The main line of argument used by Dashtakī and his followers was that Avicennian philosophers can offer true knowledge. Jalāl al-Dīn al-Dawwānī's search in the same century for an all-embracing system to reconcile Avicennian philosophy, kalām, and 'irfān must also have been related to these ongoing debates. Furthermore, one can easily see the vigor of these intellectual debates among Ottoman scholars in the many commentaries and glosses as well as treatises that were written in reply to the ideas of other schools, on topics such as being, hierarchy of subjects, and logical demonstration. Philosophical ideas were also debated in commentaries on scientific texts, as we see in the many commentaries on Naṣīr al-Dīn al-Ṭūsī's *A Memoir on the Science of Astronomy* [*al-Tadhkira fī 'ilm al-hay'a*].[81]

Conclusion

To sum up, this article has demonstrated that, prior to the 15th century, pure mathematical entities and the models based on these objects had been considered to be conjectural or imaginary. According to this view, they do not exist in external

78 Qūshjī, pp. 186-187. For the text, translation and analysis of this passage, see Ragep, "Freeing Astronomy from Philosophy", pp. 49-71, esp. pp. 61-71.

79 Ghulām Sinān, *Fatḥ al-fatḥiyya*, f. 2b.

80 Mīram Chelebī, *Risāla fī qaws quzaḥ*.

81 See, for example, Jurjānī, *Sharḥ al-Tadhkira*, Istanbul, Süleymaniye Library, Ayasofya MS 2644; Shirwānī, *Sharḥ al-Tadhkira,* Süleymaniye Library, Damad İbrahim MS 847; and Birjandī, *Sharḥ al-Tadhkira*, Rağıp Paşa Library MS 922.

reality (*bi-ḥasab al-ḥaqīqa*). But if we take mathematical models based upon mental constructs (*bi-ḥasab al-iʿtibār*) as propositions, then it is possible to form judgments in which these models correspond to reality. Therefore, they can give true (*ṣādiq*) knowledge pertaining to reality, but this knowledge provides merely the fact and not the cause. The knowledge they provide can become demonstrative knowledge (or knowledge of the reasoned fact/propter quid) if the models are shown to conform to the principles of natural philosophy. In contrast, by the 15th century if not earlier, mathematical entities and the models based on these objects were considered to exist according to what came to be known as the fact of the matter (*bi-ḥasab nafs al-amr*). According to this view, formulated by Jurjānī, Kāfiyaji and others, mathematical models, insofar as they are contained in *nafs al-amr*, can provide a true account of reality since *nafs al-amr* gurantees that the knowledge generated is *ḥaqīqī* and not merely *ṣādiq*. Though not stated explicitly by all these scholars, one implication of this, at least according to ʿAlī Qūshjī, is that a mathematical model could produce the underlying true knowledge about nature without the need for natural philosophical principles. Furthermore, inasmuch as the active intellect lost its place within virtually all epistemological systems by the 15th c., the idea that every type of *truth* is a product of sensible, conjectural, and intellectual processes assumed a dominant position in the philosophical approach of the century. As a consequence of this rise of mathematical humanism, mathematical exactitude ought to improve for the sake of certainty in mathematical knowledge. This paper argues that, more than anything else, it is this specific aspect of mathematical humanism that lies behind the remarkable advance in mathematics during the 15th century.

Historically speaking, there is no doubt that the relationship between mathematics and natural philosophy, the ontology of mathematical entities, and the truth of mathematical knowledge with regard to nature are very complicated issues. It becomes even more so when the issues examined in a specific context, in this case Islamic, are compared to parallel developments in a Western European context during the 16th and 17th centuries.[82] In fact, Descartes' search for "mathematical truth" and his attempt to relate "mathematical verity" to God resulted from a similar search.[83] In a similar vein, even the title of Newton's classic work can provide an insight into what this transformative process might have meant: *Philosophiæ Naturalis Principia Mathematica*, or *Mathematical Principles of Natural Philosophy*.

Such progress in the intellectual-philosophical realm, which had taken place in the region stretching from Istanbul to Central Asia through Iran, did not trigger a historical revolution similar to that in Western Europe. At the least, however, the

82 For a comprehensive analysis, see Feldhay, "The Use and Abuse of Mathematical Entities", pp. 80-145.

83 Hatfield, "Reason, Nature, and God in Descartes", pp. 259-287.

discourse that occurred in the Islamic world was able to produce a certain ʿAlī Qūshjī (d. 879/1474), who, representing a kind of synthesis of mathematical humanism stretching back to the Greeks and of the Ishrāqī philosophical tradition, was able to opine that the construction of mathematical knowledge pertaining to nature was possible without the need for either metaphysics or Peripatetic natural philosophy. It was Qūshjī's forcefully advanced proposal of the need to search for a new brand of natural philosophy based on mathematical principles that we believe found a receptive audience further west.[84]

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84 See note 76. Cf. Ragep, "Copernicus and His Islamic Predecessors", pp. 65-81.

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